Three-Phase, Half-Wave, Uncontrolled (Diode) Bridge Rectifier Circuits

Three-phase electricity supplies with balanced, sinusoidal voltages are widely available. It is found that the use of a three-phase rectifier system, in comparison with a single-phase system, provides smoother output voltage and higher rectifier efficiency. Also, the utilization of any supply transformers and associated equipment is better with polyphase circuits. If it is necessary to use an output filter this can be realized in a simpler and cheaper way with a polyphase rectifier.

A rectifier system with three-phase supply is illustrated by the general representation of Fig. 4.1. The instantaneous load voltage \( e_L(\omega t) \) may have an amplitude ripple but is of fixed polarity. The output current \( i_L(\omega t) \) is unidirectional but not necessarily continuous. In rectifier operation one seeks to obtain the maximum realizable average values of load voltage and current. This implies minimum load current ripple. An ideal rectifier circuit results in a continuous load current of constant amplitude and therefore constitutes an ideal dc supply.

In order to illustrate the principles of polyphase rectifier operation some simple cases of three-phase diode rectifiers are chosen. The diode elements are assumed to be ideal voltage-actuated switches having zero conducting voltage drop.

### 4.1 RESISTIVE LOAD AND IDEAL SUPPLY

Figure 4.2a shows a three-phase, half-wave uncontrolled rectifier. The supply phase voltages are presumed to remain sinusoidal at all times which implies an
ideal, impedanceless supply. The arrangement of Fig. 4.2a is electrically equivalent to three-single-phase, half-wave diode rectifiers in parallel, with a common load $R$, in which the three supply voltages are mutually displaced in time phase by $120^\circ$.

$$e_{aN} = E_m \sin \omega t$$
$$e_{bN} = E_m \sin(\omega t - 120^\circ)$$
$$e_{cN} = E_m \sin(\omega t - 240^\circ)$$

(4.1)  (4.2)  (4.3)

Since the circuit contains no inductance, it is not necessary to consider time derivatives of the currents and the operating waveforms can be deduced by inspection.

Some waveforms for steady-state operation are given in Fig. 4.2. Consider operation at a sequence of intervals of time. The terminology $t = 0^-$ means the moment of time immediately prior to $t = 0$. Similarly, $t = 0^+$ refers to the instant of time immediately following $t = 0$.

At $t = 0$, $e_{aN}$ and $e_{bN}$ are negative; $i_a = i_b = 0$. Diode $D_c$ is conducting $i_c$ so that the common cathode has the potential of point $c$. Diodes $D_a$ and $D_b$ are reverse-biased.

At $t = 0^-$, $e_{aN}$ and $e_{bN}$ remain negative. Voltage $e_{cN}$ remains more positive than $e_{aN}$ so that the common cathode still has higher potential than points $a$ and $b$. Diodes $D_a$ and $D_b$ remain in extinction.

At $t = 0^+$, $e_{aN}$ and $e_{bN}$ are positive, and $e_{cN}$ remains negative and so diode $D_c$ switches on.

At $t = 30^\circ$, $e_{aN}$ and $e_{bN}$ are more positive than $e_{cN}$. Point $a$ is higher in potential than the common cathode so that $D_a$ switches into conduction. The common cathode then has the potential of point $a$. The cathode of $D_c$ is then of higher potential than the anode so that $D_c$ switches off.

The switching sequence described above is cyclic, and a smooth transfer is effected by which the three-phase supply lines are sequentially connected to
FIG. 2 Three-phase, half-wave diode rectifier with resistive load: (a) circuit connection, (b) phase voltages at the supply, (c) load current, and (d) supply currents.
the load. Each diode is, in turn, extinguished by natural commutation due to the cycling of the supply voltages. A supply current waveform consists of the middle pieces of the corresponding positive half-wave with a conduction angle of 120°. Instantaneous supply current \( i_a(t) \) (Fig. 4.2d), for example, is defined by

\[
i_a = \frac{E_m}{R} \sin \omega t  \quad \text{for} \quad 5\pi/6, 5\pi/6 + 2\pi, ..., \quad \text{or} \quad \pi/6, \pi/6 + 2\pi, ...
\]  

(4.4)

It can be seen in Fig. 4.2c that the load current contains a ripple of three times supply frequency, and consequently, this form of rectifier is sometimes known as a three-pulse system. The average value \( I_{av} \) of the load current may be obtained by taking the average value of any 120° interval in Fig. 4.2c or of certain 60° intervals.

\[
I_{av} = \text{average value of } i_L(\omega t)
\]

\[
= \text{average value of } \frac{E_m}{R} \sin \omega t \left|_{30^\circ}^{90^\circ}\right.
\]

\[
= \frac{6}{2\pi} \int_{30^\circ}^{90^\circ} \frac{E_m}{R} \sin \omega t \, d\omega t
\]

\[
= \frac{3\sqrt{3}}{2\pi} \frac{E_m}{R}
\]

\[
= 0.827 \frac{E_m}{R}
\]  

(4.5)

Similarly, the average load voltage is seen to be

\[
E_{av} = \frac{3\sqrt{3}}{2\pi} E_m = E_{av} = I_{av}R
\]  

(4.6)

It is of interest to compare the average current value with the corresponding values 0.318 \times (E_m/R) for single-phase, half-wave operation [Eq. (2.6)] and 0.637 \times (E_m/R) for single-phase, full-wave operation (Example 2.1).

The rms value of the load current is

\[
I_L = \sqrt{\frac{6}{2\pi} \int_{30^\circ}^{90^\circ} \frac{E_m^2}{R^2} \sin^2 \omega t \, d\omega t}
\]

\[
= \frac{E_m}{R} \sqrt{\frac{4\pi + 3\sqrt{3}}{8\pi}}
\]

\[
= 0.841 \frac{E_m}{R}
\]  

(4.7)
The load dissipation is therefore

\[ P_L = I_L^2 R = \frac{E_m^2}{R} \left( \frac{4\pi + 3\sqrt{3}}{8\pi} \right) = \frac{1}{\sqrt{2}} \frac{E_m^2}{R} \]  

(4.8)

From the waveforms of Fig. 4.2d it can be seen that the rms value \( I_a \) of the supply current is given by

\[ I_a^2 = \frac{1}{2\pi} \int_0^{2\pi} \frac{E_m^2}{R^2} \sin^2 \omega t \, d\omega \]

this gives

\[ I_a = 0.485 \frac{E_m}{R} \]

\[ = I_b = I_c = I_s \]  

(4.9)

Comparison of Eqs. (4.7) and (4.9) shows that \( I_s = I_a/\sqrt{3} \).

The power factor of the three-phase, half-wave rectifier is given by

\[ PF = \frac{P_L}{3E_s I_s} \]  

(4.10)

Substituting (4.8)(4.9) into (4.10) gives

\[ PF = \frac{1/\sqrt{2} \times (E_m^2 / R)}{3(E_m/\sqrt{2}) \times 0.485 \times (E_m / R)} = 0.687 \]  

(4.11)

For the line current waveform \( i_a \) in Fig. 4.2d it is found that the Fourier coefficients of the fundamental component are given by

\[ a_1 = 0 \]

\[ b_1 = 0.471 \frac{E_m}{R} \]

\[ c_1 = 0.471 \frac{E_m}{R} \]

\[ \psi_1 = 0^\circ \]  

(4.12)

The current displacement factor \( \cos \psi_1 \) of \( i_a \) (\( \omega t \)) is therefore unity, and the power factor, Eq. (4.10), is entirely attributable to distortion effects rather than displacement effects. Part of the fundamental harmonic component \( i_{a1} \) (\( \omega t \)) is shown in Fig. 4.2d and is, because \( \psi_1 = 0^\circ \), in time phase with its phase voltage.
This was also true in the single-phase, half-wave diode rectifier with resistive load described in Sec. 2.1.2. Because \( \cos \psi_1 = 1 \), no power factor correction of this displacement factor can be realized. Any power factor improvement would have to be sought by increasing the distortion factor (i.e., by reducing the degree of distortion of the supply current waveform).

The ripple factor for the load current waveform of Fig. 4.2c is obtained by substituting values of Eqs. (4.5) and (4.7) into Eq. (2.11).

\[
RF = \sqrt{\frac{I_L^2}{I_{av}^2}} - 1
\]

\[
= \sqrt{1.034 - 1} = 0.185
\]

(4.13)

The value 0.185 in Eq. (4.13) compares very favorably with values 1.21 for the single-phase, half-wave rectifier and 0.48 for the single-phase, full-wave rectifier, with resistive load.

### 4.1.1 Worked Examples

Example 4.1 A three-phase, half-wave, uncontrolled bridge circuit with resistive load \( R = 25 \, \Omega \) is supplied from an ideal, balanced three-phase source. If the rms value of the supply voltage is 240 V, calculate (1) average power dissipation, (2) average and rms load currents, and (3) rms supply current.

The circuit diagram is shown in Fig. 4.2a. It is customary to specify a three-phase voltage supply in terms of its line-to-line rms value. In the present case the rms voltage per phase is

\[
E_s = \frac{240}{\sqrt{3}} = 138.6 \, V
\]

The peak supply voltage per-phase is therefore

\[
E_m = \sqrt{2}E_s = 240 \frac{2}{\sqrt{3}} = 196 \, V
\]

From Eq. (4.5), the average load current is

\[
I_{av} = \frac{0.827 \times 240 \times \sqrt{2}}{25 \times \sqrt{3}} = 6.48 \, A
\]

Similarly, the rms load current is given by Eq. (4.7)

\[
I_L = \frac{0.841 \times 240 \times \sqrt{2}}{25 \times \sqrt{3}} = 6.59 \, A
\]
The load power dissipation is
\[ P_L = I_L^2 = 1.086 \text{ kW} \]

In comparison, it is of interest to note that three 25-Ω resistors connected in parallel across a single-phase 240 V supply would dissipate 6.912 kW. The rms value of the supply current is obtained from Eq. (4.9).

\[ I_a = \frac{0.485 \times 240 \times \sqrt{2}}{25 \times \sqrt{3}} = 3.8 \text{ A} \]

Example 4.2 For the three-phase, half-wave rectifier of Example 4.1, calculate the input voltamperes and the power factor. Is any correction of the power factor possible by energy storage devices connected at the supply terminals?

The rms supply current per phase is, from Eq. (4.9),

\[ I_s = \frac{0.485 \times 240 \times \sqrt{2}}{25 \times \sqrt{3}} = 3.8 \text{ A} \]

The rms supply voltage per phase is given, in Example 4.1, by \( E_s = 138.6 \text{ V} \).

For a three-phase load drawing symmetrical supply currents the input voltamperes is therefore

\[ S = 3E_s I_s = 1.580 \text{ kVA} \]

The load power dissipated is all drawn through the supply terminals and is seen, from Example 2.1, to be

\[ P_L = P_s = 1.086 \text{ kW} \]

The power factor is therefore

\[ PF = \frac{P_s}{S} = \frac{1.086}{1.580} = 0.687 \]

This value of power factor is independent of \( E_s \) and \( R \) and may also be obtained directly from Eq. (4.11).

It is shown in Sec. 4.1.1 that the displacement factor \( \cos \phi_1 \) is unity. This means that the displacement angle \( \phi_1 \) is zero and that the fundamental supply current is in time phase with its respective voltage. There is therefore no quadrature component of the fundamental current and no power factor correction can be accomplished by the use of energy storage devices (which draw compensating quadrature current).

Example 4.3 For the three-phase, half-wave rectifier circuit of Fig. 4.2 calculate the displacement factor and distortion factor at the supply point and hence calculate the input power factor.
Take phase \(a\) as the reference phase. Current \(i_a(\omega t)\) is given by Eq. (4.4). Coefficients \(a_1\) and \(b_1\) of the fundamental component of the Fourier Series are given by

\[
a_1 = \frac{1}{\pi} \int_0^{2\pi} i_a(\omega t) \cos \omega t \, d\omega t = \frac{E_m}{\pi R} \int_{-\pi}^{\pi} \sin \omega t \cos \omega t \, d\omega t = \frac{E_m}{4\pi R} \left[ -\cos 2\omega t \right]_{-\pi/2}^{3\pi/2} = 0
\]

\[
b_1 = \frac{1}{\pi} \int_0^{2\pi} i_a(\omega t) \sin \omega t \, d\omega t = \frac{E_m}{\pi R} \int_{-\pi}^{\pi} \sin^2 \omega t \, d\omega t = \frac{E_m}{2\pi R} \left[ \omega t - \frac{1}{2} \sin 2\omega t \right]_{-\pi/2}^{3\pi/2} = \frac{E_m}{2\pi R} \left[ \frac{2\pi}{3} \times \frac{\sqrt{3}}{2} \right] = 0.471 \frac{E_m}{R}
\]

The peak magnitude \(c_1\) of the fundamental component \(i_{a1}(\omega t)\) of \(i_a(\omega t)\) is therefore

\[
c_1 = \sqrt{a_1^2 + b_1^2} = 0.471 \frac{E_m}{R}
\]

Since \(a_1 = 0\), the phase angle \(\psi_1\) is also zero

\[
\psi_1 = \tan^{-1} \left( \frac{a_1}{b_1} \right) = 0
\]

Displacement factor = \(\cos \psi_1 = 1.0\)

Distortion factor = \(\frac{I_{a1}}{I_a}\)

The rms value of the fundamental supply current component is given by

\[
I_{a1} = \frac{0.471 \frac{E_m}{R}}{\sqrt{2}} = 0.333 \frac{E_m}{R}
\]
The rms value of the total supply current per phase is given by Eq. (4.8)

\[ I_a = 0.485 \frac{E_m}{R} \]

Therefore, the distortion factor is given by

\[ \text{Distortion factor} = \frac{0.333}{0.485} = 0.687 \]

The system power factor is therefore

\[ PF = (\text{distortion factor})(\text{displacement factor}) = 0.687 \]

which agrees with Example 4.2

### 4.2 RESISTIVE LOAD WITH TRANSFORMER COUPLED SUPPLY

In some applications a three-phase rectifier circuit is fed from the star-connected secondary windings of a transformer, as in Fig. 4.3.

The ratio load power/transformer secondary voltamperes is sometimes referred to as the secondary utilization factor. But, for a star-star-connected transformer, the primary voltamperes is equal to the secondary voltamperes so that the above ratio is then more familiarly seen as the power factor of the trans-

![Fig. 3](image-url)  

Three-phase, half-wave diode rectifier fed from transformer source.
former–rectifier–load combination. The primary windings of a rectifier transformer also may be connected in delta to provide a path for triplen harmonic currents. Many different transformer connections have been used, and they are characterized by particular waveforms, requiring relevant ratings of the transformer windings. For example, two three-phase, half-wave secondary connections may be combined via an interphase transformer winding, as shown in Fig. 4.4. This results in a performance known as *six-pulse operation*, described in Chapter 6.

The great disadvantage of the circuit of Fig. 4.2a is that a direct current component is drawn through each supply line. With transformer coupling (Fig. 4.3) the dc components in the secondary windings could saturate the transformer cores. This may be avoided by the use of a zigzag connection in which the dc magnetomotive forces of two secondary windings on the same core cancel out.

![Fig. 4 Double-star or interphase transformer connection to produce six-pulse operation.](image-url)
The half-wave bridge is of limited practical value in industry but is very useful as an educational aid in understanding the basic operation of polyphase bridge circuits.

### 4.3 HIGHLY INDUCTIVE LOAD AND IDEAL SUPPLY

If the load resistor $R$ in the three-phase, half-wave circuit, has a highly inductive load impedance $L$ in series (Fig. 4.5a), this smoothing reactor absorbs most of the load voltage ripple. Although this ripple can never be entirely eliminated, the load current and the voltage across the load resistor are virtually constant (Fig. 4.5d). The instantaneous load voltage retains its segmented sinusoidal form (Fig. 4.5c), as with resistive load.

For the load branch

$$\begin{align*}
e_L(\omega t) &= e_g(\omega t) + e_r(\omega t) \\
&= i_L R + L \frac{di_L}{dt} \quad (4.14)
\end{align*}$$

It is seen from Fig. 4.5c that

$$\begin{align*}
e_L(\omega t) &= E_m \sin \omega t \begin{bmatrix} 150^\circ + E_m \sin(\omega t - 120) \\ 30^\circ + E_m \sin(\omega t - 240) \end{bmatrix} \\
&= 0, 270^\circ \quad \text{for} \quad 30^\circ, 360^\circ
\end{align*} \quad (4.15)$$

With a large series inductance the load current becomes very smooth with an almost constant value $I$ given by

$$I_{av} = \frac{E_m}{R} = \frac{3\sqrt{3} E_m}{2\pi R} = \frac{E_{av}}{R} \quad (4.16)$$

which is identical to Eq. (4.5) for resistive load. But the instantaneous load voltage $e_L(\omega t)$ may be thought of as consisting of its average value $E_{av}$ plus a ripple component (i.e., a nonsinusoidal alternating component), $e_r(\omega t)$:

$$e_L(\omega t) = E_{av} + e_r(\omega t) \quad (4.17)$$

Now with a constant load current (Fig. 4.5d), its instantaneous, average, rms, and peak values are identical.

$$i_L(\omega t) = I_{av} = I_L = \frac{E_{av}}{R} \quad (4.18)$$

Comparing Eqs. (4.14) and (4.17), noting that $i_L = I_{av}$ shows that
FIG. 5 Three-phase, half-wave diode rectifier with highly inductive load: (a) circuit connection, (b) supply phase voltages, (c) load voltage, (d) load current, and (e) supply currents.
\[ e_r(\omega t) = L \frac{di}{dt} = \alpha L \frac{di}{d\omega t} \] (4.19)

The time variation of \( e_r(\omega t) \) in Fig. 4.5c, above and below the \( E_{av} \) value, is the component \( e_r(\omega t) \), and this falls entirely on the inductor \( L \). Instantaneous ripple voltage \( e_r(\omega t) \) can also be expressed in terms of the flux \( \varphi \) associated with the inductor

\[ e_r(\omega t) = N \frac{d\varphi}{dt} = L \frac{di}{dt} \] (4.20)

The flux is therefore given by

\[ \varphi = \frac{1}{N} \int e_r \, dt \] (4.21)

which is the net area shaded in Fig. 4.5d. The time average values of the inductor voltage \( e_r(\omega t) \) and flux \( \varphi(\omega t) \) are zero so that the two respective shaded portions represent equal areas and equal values in volt-seconds.

Note that the instantaneous inductor voltage \( e_r(\omega t) \) is only zero at six instants in each supply voltage cycle and not at every instant as would be the case if a pure direct current was injected into a pure inductor.

As with resistive load, the supply current pulses (Fig. 4.5e) have a conduction angle of 120°. The rms value \( I_a \) of the supply currents is related to the average load current by

\[ I_a = \sqrt{\frac{1}{2\pi} \int_{\omega t}^{\omega t + 180} I^2 \, d\omega t} = \frac{I_m}{\sqrt{3}} \] (4.22)

Combining Eqs. (4.5) and (4.22) gives

\[ I_a = \frac{3}{2\pi} \frac{E_m}{R} = 0.477 \frac{E_m}{R} \] (4.23)

which is about 2% lower than the value \( 0.485E_m/R \) for resistive load, Eq. (4.9). All the average power dissipation in the circuit is presumed to occur in the load resistor. Therefore,

\[ P_L = I_a^2 R = I_m^2 R = \left( \frac{3\sqrt{3}}{2\pi} \right) \frac{E_m^2}{R} = 0.684 \frac{E_m^2}{R} \] (4.24)

The circuit power factor is therefore given by
\[
P F = \frac{\left(3\sqrt{3}/2\pi\right)E_n^2}{3(E_m/\sqrt{2})(3/2\pi)(E_m/R)} = 0.675
\]

which is marginally lower than the corresponding value 0.687, Eq. (4.10), with resistive load. It should be noted that the value of the power factor is independent of voltage level and of load impedance values. A comparative summary of some of the properties of the three-phase, half-wave, uncontrolled bridge is given in the top half of Table 4.1

<table>
<thead>
<tr>
<th></th>
<th>Resistive load</th>
<th>Highly inductive load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three-pulse (half-wave)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>operation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average load current</td>
<td>(0.27\frac{E_m}{R})</td>
<td>(0.27\frac{E_m}{R})</td>
</tr>
<tr>
<td>RMS load current</td>
<td>(0.41\frac{E_m}{R})</td>
<td>(0.27\frac{E_m}{R})</td>
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<tr>
<td>Load power</td>
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<td>(0.84\frac{E_m}{R})</td>
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<tr>
<td>RMS supply current</td>
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<td>Power factor</td>
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<td>0</td>
</tr>
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<td>Six-pulse (full-wave)</td>
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<td></td>
</tr>
<tr>
<td>operation</td>
<td></td>
<td></td>
</tr>
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<td>Average load current</td>
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<td>(0.55\left(\sqrt{3}\frac{E_m}{R}\right))</td>
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<td>(0.55\left(\sqrt{3}\frac{E_m}{R}\right))</td>
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</tr>
<tr>
<td>Load current</td>
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<td>0</td>
</tr>
</tbody>
</table>
4.3.1 Worked Examples

Example 4.4 A three-phase, half-wave diode bridge is supplied from an ideal three-phase voltage source. Calculate the ripple factor of the load voltage, the supply current and load current with (1) resistive load and (2) highly inductive load.

The load voltage waveform for an ideal bridge with ideal supply voltages is shown by Fig. 4.5c for both resistive and inductive loads. For this waveform, from Eq. (4.16),

\[ E_{a.v} = I_{a.v} R = 0.827 \frac{E_m}{R} \]

Also, from Eq. (4.6), the rms value \( E_L \) of the load voltage is

\[ E_L = 0.841 \frac{E_m}{R} \]

The ripple factor of the load voltage is therefore

\[ RF = \sqrt{\left( \frac{I_{a.v}}{I_{a.v}} \right)^2} - 1 = 0.185 \]

This low value is consistent with the relatively smooth waveform of Fig. 4.5c.

1. For resistive load, the load current ripple factor is also equal to 0.185.

For the supply current with resistive load, the rms value, Eq. (4.8) is

\[ I_s = 0.485 \frac{E_m}{R} \]

The average value of the supply current waveform (Fig. 4.2d), is one third the average value of the load current waveform (Fig. 4.2c). From Eq. (4.5),

\[ I_{a.v} = \frac{0.827 E_m}{3 R} = 0.276 \frac{E_m}{R} \]

With resistive load the supply current ripple factor is therefore

\[ RF = \sqrt{\left( \frac{I_{a.v}}{I_{a.v}} \right)^2} - 1 = 1.44 \]

This high value is consistent with the high value \( (0.471E_m/R) \) of the fundamental ac component of \( i_s (\omega t) \) shown in Fig. 4.2d.
2. With highly inductive load (Fig. 4.5), the rms supply current [Eq. (4.23)] is

\[ I_s = 0.477 \frac{E_m}{R} \]

The average value of the supply current is one third the average value of the load current [Eq. (4.16)]:

\[ I_{av} = \frac{\sqrt{3}}{2\pi} \frac{E_m}{R} = \frac{E_{av}}{3R} = 0.276 \frac{E_m}{R} \]

With highly inductive load the supply current ripple factor is therefore

\[ RF = \sqrt{0.477^2 - 0.276} = 1.41 \]

The effect of the load inductance is seen to reduce the supply current ripple factor only marginally, because the discontinuous current waveform is mainly attributable to natural switching by the diode elements.

Example 4.5 A three-phase, half-wave diode bridge with a highly inductive load is supplied from an ideal, three-phase source. Determine the waveform of a diode voltage and calculate the diode rms voltage rating.

Consider the circuit of Fig. 4.5a. During the conduction of current \( i_a (\omega t) \), the voltage drop on diode \( D_a \) is ideally zero. While \( D_a \) is in extinction, its anode is held at potential \( e_{aN} \) and its cathode is held at either \( e_{bN} \) or \( e_{cN} \) depending on whether \( D_b \) or \( D_c \) is conducting, respectively. In Fig. 4.5 the following voltages pertain to parts of the cycle

\[
\begin{align*}
0 < \omega t < 30^\circ & \quad e_{D_a} = e_{aN} - e_{cN} = e_{ac} \\
30^\circ < \omega t < 150^\circ & \quad e_{D_a} = 0 \\
150^\circ < \omega t < 270^\circ & \quad e_{D_a} = e_{aN} - e_{bN} = e_{ab} \\
270^\circ < \omega t < 360^\circ & \quad e_{D_a} = e_{aN} - e_{cN} = e_{ac}
\end{align*}
\]

The line voltages \( e_{ac} \) and \( e_{ab} \) are \( \sqrt{3} \) times the magnitude values of the phase voltages and result in the diode anode-cathode voltage waveform shown in Fig. 4.6, which can be deduced from the three-phase line voltage waveforms.

The rms value of waveform \( E_{D_a} (\omega t) \) in Fig. 4.6c may be obtained from

\[
E_{D_a} = \sqrt{\frac{2}{2\pi} \int_{150^\circ}^{360^\circ} \left[ \sqrt{3} E_m \sin(\omega t - 30^\circ) \right]^2 d\omega t}
\]

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FIG. 6 Waveforms for operation of a three-phase, half-wave diode rectifier with highly inductive load.

\[
E_{D_a}^2 = \frac{3}{\pi} E_m^2 \left[ \frac{\omega - 30^\circ}{2} - \frac{\sin (2(\omega - 30^\circ))}{4} \right]_{150^\circ}^{270^\circ}
\]

Using radian values for the angle terms, this becomes

\[
E_{D_a}^2 = \frac{3}{\pi} E_m^2 \left[ \frac{\sqrt{3}}{2} - \frac{1}{4} \left( \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right) \right]
\]

From which,

\[
E_{D_a} = 0.64 E_m
\]

This value is sketched in Fig. 4.6c and looks reasonable. For design purposes a more relevant property of the diode is the peak reverse voltage (PRV) that it is
required to withstand. In this case, it is clear that the PRV has a magnitude of \( \sqrt{3}E_m \). The reverse (i.e., cathode–anode) voltage waveform of diode \( D_a \) will be the reverse of waveform \( E_{D_a} \) in Fig. 4.6.

Example 4.6  A three-phase, half-wave rectifier is fed from an ideal three-phase supply of value 400 V at 50 Hz. The load current is maintained constant at 50 A by the use of a suitable load side inductor. If the conducting voltage drop of the diodes is 1 V, calculate the required diode ratings and the value of the load resistor.

The diode peak reverse voltage rating is the peak value of the line voltage.

\[
E_{(PRV)} = 400 \sqrt{2} = 566 \text{ V}
\]

The average load current is specified as

\[
I_{av} = 50 \text{ A}
\]

The supply current pulses in Fig. 4.5 have an rms value given by Eq. (4.22)

\[
I_s = \frac{I_{av}}{\sqrt{3}} = 28.9 \text{ A}
\]

The conducting voltage drop on the bridge diodes has no effect on their voltage or current ratings.

Now the average load voltage is given, from Eq. (4.16), by

\[
E_{av} = \frac{3\sqrt{3}}{2\pi}E_m
\]

where \( E_m \) is the peak phase voltage. In this case \( E_m = 400 \sqrt{2}/\sqrt{3} \) so that

\[
E_{av} = \frac{3}{2\pi}400\sqrt{2}/\sqrt{3} - 1 = 269 \text{ V}
\]

From Eq. (4.16),

\[
R = \frac{E_{av}}{I_{av}} = \frac{269}{50} = 5.38 \Omega
\]

4.4 HIGHLY INDUCTIVE LOAD IN THE PRESENCE OF SUPPLY IMPEDANCE

An electrical power supply is not usually a perfect voltage source because it contains series impedance. The action of drawing current from the supply into a resistive or inductive load causes the supply voltage at the terminals to reduce
below its no-load value. In public electricity supply undertakings, the generator voltage level is usually automatically boosted to provide constant voltage at a consumer’s terminals when load current is drawn from the supply. The series impedance of an electricity supply system is usually resistive-inductive, being created by transformers, cables, and transmission supply lines. In transformer-supplied bridge circuits the supply inductance is mainly the transformer leakage inductance.

The magnitude of the supply inductance is typically such that not more than about 5% reduction would occur in an unregulated supply voltage at full-load current. Because of this inductance the instantaneous commutation of current from one diode to another that occurs in resistive circuits, described in Sec. 4.1.1, for example, cannot occur. When switching closure occurs in an open inductive circuit a definite time is required for a current to build up from zero to its final steady-state value. The instantaneous transitions in the value of the supply currents in Fig. 4.2, for example, no longer take place.

Consider operation of the half-wave diode bridge (Fig. 4.7). The balanced sinusoidal voltages of the generator are given by $e_{AN}$, $e_{BN}$, and $e_{CN}$, where

\[
e_{AN} = E_m \sin \omega t \\
e_{BN} = E_M \sin (\omega t - 120^\circ) \\
e_{CN} = E_m \sin (\omega t - 240^\circ)
\]  

(4.26)  
(4.27)  
(4.28)

FIG. 7  Three-phase, half-wave diode rectifier with highly inductive load inductance.
When each supply line contains an effective series inductance $L_s$; Fig. 4.7, the bridge terminal voltages $e_{AN}$, $e_{BN}$, and $e_{CN}$ do not remain sinusoidal on load but are given by

$$e_{AN} = e_{AN} - L_s \frac{di_a}{dt}$$  \hspace{1cm} (4.29)

$$e_{AN} = e_{BN} - L_s \frac{di_b}{dt}$$  \hspace{1cm} (4.30)

$$e_{AN} = e_{CN} - L_s \frac{di_c}{dt}$$  \hspace{1cm} (4.31)

Compared with operation with an ideal supply, as in Fig. 4.5a, the waveforms of both the terminal voltage and current and the load voltage are affected by the presence of supply reactance. Now the magnitude of the supply inductance $L_s$ is usually small compared with the value of the load inductance $L$. The presence of supply inductance therefore does not significantly affect the magnitude $I_{av}$ of the load current nor the maximum value $I_{av}$ and average value $I_{av}/3$ of the supply currents in Fig. 4.5. The magnitude of the load current at fixed supply voltage is determined almost entirely by the value of the load resistance because the load inductor offers no net impedance to the (hopefully predominant) direct current component. It is also of interest that the waveform of the supply currents, at fixed supply voltage, varies according to the level of the load current and the value of the supply inductance. Several different modes of supply current behavior are identifiable, depending on the particular application.

Figure 4.8 gives some detail of the waveforms due to operation of the circuit of Fig. 4.7. Diode $D_a$ carries the rectified current of peak magnitude $I_{av}$ up to a point $x$. Since the anode voltages $e_{AN}$ and $e_{BN}$ of $D_a$ and $D_b$ are then equal, diode $D_b$ starts to conduct current $i_b$. Due to the effect of the supply inductance, the current $i_b$ cannot extinguish immediately as it does with ideal supply, shown in Fig. 4.8b. Due to the supply line inductance, diodes $D_a$ and $D_b$ then conduct simultaneously, which short circuits terminals a and b of the supply. During this interval of simultaneous conduction, called the overlap period or commutation angle $u$, the common cathode has a potential $e_{AN} + e_{BN}/2$. In the overlap period $xy$, $e_{BN}$ is greater than $e_{AN}$ and the difference voltage can be considered to cause a circulating current in loop aPbNa (Fig. 4.7), which increases $i_b$ and diminishes $i_a$. When current $i_a$ falls below its holding value diode $D$ switches into extinction and the load voltage then jumps to the corresponding point $z$ on wave $e_{BN}$. Each supply current has the characteristic waveform of Fig. 4.8c, but the load current is continuous and smooth at the value $I_{av}$ (Fig. 4.8e). Because the presence of supply inductance does not affect the maximum value $I_{av}$ nor the
Fig. 8  Waveforms for operation of a three-phase, half-wave diode rectifier with highly inductive load and supply inductance: (a) load voltage, (b) supply line current (with ideal supply), (c) and (d) supply line currents $i_a(\omega t)$ and $i_b(\omega t)$, and (e) load current.
average value \( I_{av} / 3 \) of the supply current the total area under the current pulse (Fig. 4.8c) is unchanged, compared with ideal supply.

Comparison of Fig. 4.8a with Fig. 4.5c shows, however, that an effect of supply reactance is to reduce the average value \( E_{av} \) of the load voltage. From Fig. 4.8a it is seen that, with overlap angle \( u \),

\[
E_{av} = \frac{3}{2\pi} \left[ \int_{\alpha}^{\alpha+30^\circ} \frac{e_{AN} + e_{CN}}{2} \, dt + \int_{\alpha+30^\circ}^{150^\circ} e_{AN} \, dt \right]
\]

\[
= \frac{3\sqrt{3}}{2\pi} E_m \cos^2 \frac{u}{2}
\]

\[
= \frac{3\sqrt{3}E_m}{4\pi} (1 + \cos u)
\]

This may also be expressed

\[
E_{av} = E_{avo} \cos^2 \frac{u}{2} = \frac{E_{avo}}{2} (1 + \cos u)
\]

where \( E_{avo} \) is the average load voltage with zero overlap, or ideal ac supply, defined in Eq. (4.7).

In the circuit of Fig. 4.7, during the overlap created by the simultaneous conduction of diodes \( D_a \) and \( D_b \), there is no current in supply line \( c \), and

\[
e_{AN} - L_s \frac{di_s}{dt} = e_{BV} - L_s \frac{di_s}{dt}
\]

But the load current \( I_{av} = i_a + i_b \) is not affected by the presence of \( L_s \). Therefore,

\[
e_{AN} - L_s \frac{di}{dt} = e_{BV} - L_s \frac{di}{dt} (i_a - i_b)
\]

and

\[
e_{AN} - e_{BV} = 2L_s \frac{di}{dt}
\]

Substituting Eqs. (4.29) and (4.30) into Eq. (4.36) and integrating from \( 150^\circ \) to \( 150^\circ + u \), noting that \( i_a = I_{av} \) at \( \omega t = 150^\circ \), gives

\[
\cos u = 1 - \frac{2\omega L_s I_{av}}{\sqrt{3}E_m}
\]

Combining Eqs. (4.6) and (4.37) permits \( u \) to be expressed in terms of impedance parameters, utilizing the fact that \( I_{av} = E_{avo}/R \).
Combining Eqs. (4.35) and (4.38) permits \( \cos u \) to be expressed in terms of impedance parameters

\[
\cos u = 1 - \frac{3\omega L_s I_{av}}{\pi E_{av}} = 1 - \frac{3}{\pi} \frac{\omega L_s}{R} \frac{E_{av}}{E_{av0}}
\]  

(4.38)

Provided that the load inductance \( L \) is large, the actual value of \( L \) does not occur in the relevant circuit equations. With a good electrical supply the ratio \( \omega L_s/R \) is about 0.05 at full load and the value of \( u \) is then about 18°. For a poor (i.e., relatively high inductance) supply, or with reduced load resistance such that \( \omega L_s/R = 0.2 \), then \( u \) is about 34°. A value \( u = 18° \) results in a reduction of \( E_{av} \) of less than 3%, while \( u = 34° \) results in about 9% reduction. The reduction of average load voltage can be expressed in terms of impedance parameters by combining Eqs. (4.33) and (4.39).

\[
E_{av} = \frac{E_{av0}}{1 + \left( \frac{3\omega L_s}{2\pi R} \right)}
\]  

(4.40)

The supply inductance is found to modify the previously appropriate expression Eq. (4.22), for rms supply current to

\[
I_a = \frac{I_{av}}{\sqrt{3}} \sqrt{1 - 3\psi(u)}
\]  

(4.41)

where

\[
\psi(u) = \frac{(2 + \cos u)\sin u - (1 + 2\cos u)u}{2\pi(1 - \cos u)^2}
\]  

(4.42)

Function \( \psi(u) \) varies almost linearly with \( u \) for values up to \( u = 60° \). At \( u = 34° \), for example, the effect of supply reactance is found to reduce \( I_a \) by about 3.5%.

The effect of gradually increased overlap, with fixed supply voltage, is demonstrated sequentially in Figs. 4.9–4.15. Note that for values of supply inductance such that \( u > 90° \), Figs. 4.13–4.15, the load resistance is here modified to give the same peak value of supply current. For values of \( u < 90° \) the performance is usually described as mode I operation. With \( u > 90° \) the load phase voltages become discontinuous and the performance is referred to as mode II operation.
It is seen that the conduction angle of the supply currents progressively increases with overlap.

The boundary between mode I operation and mode II operation occurs at \( u = 90^\circ \). Under that condition \( \cos u \) is zero and it is seen from equation (4.37) that

\[
I_{av} \bigg|_{u=90^\circ} = \frac{\sqrt{3}E_m}{2\omega L_s}
\]  
(4.43)

But from Fig. 4.7, the right hand side of Eq. (4.43) is seen to be the peak value of the short-circuit current \( I_{sc} \) in (say) loop APBN.

Therefore,

\[
\hat{I}_{sc} = \frac{\sqrt{3}E_m}{2\omega L_s} = I_{av} \bigg|_{u=90^\circ}
\]  
(4.44)

Combining Eqs. (4.37), (4.43), and (4.44) results in

\[
\frac{I_{av}}{I_{sc}} = 1 - \cos u
\]  
(4.45)

The short circuit current can be expressed in terms of the average load voltage by eliminating \( \cos u \) between Eqs. (4.38), (4.40), and (4.45).

\[
\frac{E_{av}}{E_{avo}} = 1 - \frac{I_{av}}{2I_{sc}}
\]  
(4.46)

For \( u > 90^\circ \), which occurs in mode II operation, \( I_{av} > \hat{I}_{sc} \) and the average load voltage, \( E_{av} \) becomes less than one half of the value \( E_{avo} \) with ideal supply.

All the energy dissipation in the circuit of Fig. 4.7 is presumed to occur in the load resistor \( R \). The power rating of the circuit in mode I is given in terms of the constant value \( I_{av} \) of the load current.

\[
P = \frac{I_{av}^2}{2}R
\]  
(4.47)

In mode I the supply current has the rms value denoted in Eq. (4.41). The supply voltage \( e_{av} (\omega t) \) is seen from Figs. 4.10–12 to be given by

\[
e_{av}(\omega t) = E_m \sin \omega t \begin{cases} 30^\circ, 150^\circ, 360^\circ \\
0, u+30^\circ, u+150^\circ \\
\frac{1}{2}(e_{AN} + e_{CN}) \\
\frac{1}{2}(e_{AN} + e_{BN}) \\
\end{cases} \begin{cases} u+30^\circ \\
\frac{1}{2}(e_{AN} + e_{BN}) \\
150^\circ \\
\end{cases}
\]  
(4.48)
FIG. 9 Waveforms for three-phase, half-wave diode bridge with highly inductive load. Ideal supply $u = 0$. 
FIG. 10 Waveforms for three-phase, half-wave diode bridge with highly inductive load. Mode I, $\mu = 30^\circ$, ideal supply.
Fig. 11 Waveforms for three-phase, half-wave diode bridge with highly inductive load. Mode 1, $\mu = 60^\circ$, ideal supply.
FIG. 12  Waveforms for three-phase, half-wave, diode bridge with highly inductive load. Limit mode I, $\mu = 90^\circ$, ideal supply.
Fig. 13 Waveforms for three-phase, half-wave diode bridge with highly inductive load. Mode II, $\mu = 105^\circ$, ideal supply.
FIG. 14  Waveforms for three-phase, half-wave diode bridge with highly inductive load. Mode II, $\mu = 120^\circ$, ideal supply.
FIG. 15 Waveforms for three-phase, half-wave diode bridge with highly inductive load. \( \mu = 150^\circ \), ideal supply.
which has the rms value $E_{aN}$, where

$$E_{aN}^2 = \frac{1}{2\pi} \int_0^{2\pi} e_{aN}^2 \, d\omega t \tag{4.49}$$

The substitution of Eq. (4.48) into Eq. (4.49) gives

$$E_{aN} = E_m \sqrt{\frac{1}{2\pi} \left( \pi - \frac{3u}{4} + \frac{1}{4} \sin 2u \right)} \tag{4.50}$$

The power factor of the three-phase, half-wave bridge is given by

$$PF = \frac{P}{3E_{aN}I_a} \tag{4.51}$$

Supply reactance causes both the rms voltage $E_{aN}$ and rms current $I_a$ to be reduced below their respective levels with ideal supply. The power factor is therefore increased.

### 4.4.1 Worked Examples

Example 4.7 A three-phase, half-wave uncontrolled bridge circuit transfers energy from a three-phase supply to a highly inductive load consisting of a resistor $R$ in series with inductor $L$. Each supply line may be considered to have a series inductance $L_s$. Show that the average load voltage is given by

$$E_{av} = \frac{3\sqrt{3}E_m}{2} - \frac{3\omega L_s I_m}{2\pi}$$

where $E_m$ is the peak phase voltage.

The circuit diagram is shown in Fig. 4.7. From Eq. (4.33)

$$E_{av} = \frac{E_{m}(1 + \cos u)}{2} = \frac{3\sqrt{3}E_m}{4\pi} (1 + \cos u)$$

Substituting $\cos u$ from Eq. (4.37) gives

$$E_{av} = \frac{3\sqrt{3}E_m}{4\pi} \left( 2 - \frac{2L_s \omega I_m}{\sqrt{3}E_m} \right)$$

Therefore,

$$E_{av} = \frac{3\sqrt{3}E_m}{2\pi} - \frac{3L_s \omega I_m}{2\pi} = \frac{3}{2\pi} \left( \sqrt{3}E_m - \omega L_s I_m \right)$$

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The final form of $E_{av}$ above shows that this incorporates the peak line-to-line voltage $\sqrt{3} E_m$ less the line voltage drop due to the supply reactance.

Example 4.8 A three-phase, half-wave diode bridge supplies power to a load consisting of resistor $R$ and series inductor $L$. Each phase of the supply has a series inductance $L_s$ where $L_s << L$. Sketch waveforms of the per-phase voltage and current of the supply for mode I operation when $\mu = 30^\circ$. Derive an expression for the instantaneous supply current $i_a(\omega t)$ for the overlap period $150^\circ < \omega t < 150^\circ + \mu$.

Waveforms of $e_{AN}(\omega t)$ and $i_a(\omega t)$ for $\mu = 30^\circ$ are given in Figs. 4.8 and 4.10. Instantaneous current $i_a(\omega t)$ is defined by Eq. (4.36),

$$e_{AN} - e_{BN} = 2L_s \frac{di_a}{dt}$$

where

$$e_{AN} = E_m \sin \omega t$$

$$e_{BN} = E_m \sin \left(\omega t - \frac{2\pi}{3}\right)$$

Therefore,

$$\frac{di_a}{dt} = \frac{e_{AN} - e_{BN}}{2L_s} = \frac{E_m}{2L_s} 2 \sin \frac{\pi}{3} \cos \left(\omega t - \frac{\pi}{3}\right)$$

$$= \frac{\sqrt{3}}{2} \frac{E_m}{L_s} \cos \left(\omega t - \frac{\pi}{3}\right)$$

Integrating both sides of the differential equation gives

$$i_a(t) = \frac{\sqrt{3}}{2} \frac{E_m}{\omega L_s} \sin \left(\omega t - \frac{\pi}{3}\right) + K$$

where $K$ is a constant of integration.

Now (1) at $\omega t = 150^\circ$, $i_a = I_{av}$; (2) at $\omega t = 150^\circ + \mu$, $i_a = 0$.

Under condition (1),

$$K = I_{av} - \frac{\sqrt{3}}{2} \frac{E_m}{\omega L_s} = I_{av} - \dot{I}_{sc}$$

which is negative because $\dot{I}_{sc} > I_{av}$ for mode I operation.

Under condition (2),

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\[ K = -\frac{\sqrt{3} E_m}{2 \omega L_s} \cos u = -\hat{I}_m \cos u \]

Since \( u = 30^\circ \),

\[ K = -\frac{\sqrt{3}}{2} \hat{I}_m \]

\[ = -\frac{3 E_m}{4 \omega L_s} \]

Equating these two values of \( K \) between the two consistent conditions shows that

\[ I_m = \frac{\sqrt{3} E_m}{2 \omega L_s} \left( 1 - \frac{\sqrt{3}}{2} \right) \]

\[ = \hat{I}_m \left( 1 - \frac{\sqrt{3}}{2} \right) \]

which is seen to be consistent with Eq. (4.45)

Using the value of \( K \) from condition (2) in the equation for \( i_m \) gives

\[ i_m = \frac{\sqrt{3}}{2} \frac{E_m}{\omega L_s} \left[ \sin \left( \omega t - \frac{\pi}{3} \right) - \frac{\sqrt{3}}{2} \right] \]

for \( 150^\circ < \omega t < 150^\circ + u \)

for \( 150^\circ < \omega t < 150^\circ + u \)

Example 4.9 A three-phase, half-wave diode rectifier has a load resistance \( R = 10 \, \Omega \) in series with a large inductor. Each supply line contains a series inductance \( L_s \) such that the inductive reactance in the line is 10% of the load resistance. The generator three-phase voltages have an rms line value of 400 V. Calculate the power factor of operation and compare this with the case of ideal supply.

The equivalent circuit is given in Fig. 4.7. The first step of the solution is to calculate the overlap angle \( u \). Since

\[ \frac{\omega L_s}{R} = 0.1 \]

then from Eq. (4.39),

\[ \cos u = 0.9086 \]

Therefore, \( u = 24.7^\circ = 0.431 \, \text{rad} \), which is mode I operation. Function \( \psi(u) \), Eq. (4.42), is found to have the value
\[
\psi(u) = \frac{(2.9086)(0.418) - (1 + 1.817)(0.431)}{2\pi(0.0915)^2} = \frac{1.2158 - 1.2141}{0.0526} = \frac{0.0017}{0.0526} = 0.032
\]

The average load voltage in the presence of \(L\) is given by Eq. (4.32) in which \(E_m\) is the peak phase voltage. In the present case,

\[
E_m = \frac{400\sqrt{3}}{\sqrt{3}} = 326.6\text{V}
\]

Therefore,

\[
E_{av} = \frac{3\sqrt{3}E_m(1 + \cos u)}{4\pi} = 135(1.9085) = 257.7\text{V}
\]

This compares with the value \(E_{av,o} = (3\sqrt{3}/2\pi)E_m = 270\text{V}\) obtainable with an ideal supply.

The average load current is unchanged by the presence of the supply inductance.

\[
I_{av} = \frac{E_{av}}{R} = \frac{270}{10} = 27\text{A}
\]

From Eq. (4.42) the rms supply current is

\[
I_a = \frac{I_{av}}{\sqrt{3}} \sqrt{1 - 3\psi(u)} = \frac{27}{\sqrt{3}} \sqrt{1 - 0.096} = 14.82\text{A}
\]

This compares with the value \(I_a = I_{av}/\sqrt{3} = 15.6\text{A}\) with ideal supply. The power into the bridge circuit is presumed to be dissipated entirely in the load resistor:

\[
P = I_a^2R = (27)^2 10 = 7290\text{W}
\]

The power factor, seen from the supply terminals, is given by Eq. (4.51), which incorporates the rms value \(E_{av}\) of the terminal voltage. For a value \(u = 24.7^\circ\)
the waveform $e_{an}(\omega t)$ is very similar to that given in Fig. 4.10a and is defined by Eq. (4.48). Inspection of the more detailed diagram (Fig. 4.8a) shows that

$$e_{an}(\omega t) = \frac{E_m}{2} \sin(\omega t + 60^\circ) \left[ 30^\circ + u \right] + \frac{E_m}{2} \sin(\omega t - 60^\circ) \left[ 150^\circ + u \right] + \frac{E_m}{2} \sin \omega t \left[ 30^\circ, 150^\circ, 360^\circ \right] + \frac{E_m}{2} \sin \omega t \left[ 0, 30^\circ + u, 150^\circ + u \right]$$

Therefore,

$$E_i^2 = \frac{E_m^2}{2\pi} \left[ \frac{u}{4} \right] \left[ \frac{1}{4} \int_{30^\circ}^{150^\circ} \sin^2(\omega t + 60^\circ) \, d\omega t + \frac{1}{4} \int_{150^\circ}^{30^\circ} \sin^2(\omega t - 60^\circ) \, d\omega t \right] + \frac{1}{4} \left( \frac{\omega t}{2} - \frac{\sin^2(\omega t - 60^\circ)}{4} \right) _{150^\circ + u} ^{150^\circ} + \frac{1}{4} \left( \frac{\omega t}{2} - \frac{\sin 2\omega t}{4} \right) _{0, 30^\circ + u, 150^\circ + u} ^{30^\circ, 150^\circ, 360^\circ}$$

$$= \frac{E_m^2}{2\pi} \left[ \frac{u}{4} \left( \frac{1}{4} \sin(\pi/2) + \frac{1}{4} \sin(\pi - 2u) \right) + \frac{1}{4} \left( \frac{1}{4} \sin(\pi/2) - \frac{1}{4} \sin(\pi - 2u) \right) + \frac{2\pi - 2u}{2} - \frac{1}{4} \frac{\sqrt{3}}{2} - 0 \right]$$

$$-\frac{\sqrt{3}}{2} \sin(60^\circ + 2u) + 0 - \sin(300^\circ + 2u)$$

$$= \frac{E_m^2}{2\pi} \left[ \frac{1}{4} \left( 1 + \sin(\pi/2) - 2u \right) + \frac{1}{4} \left( 1 - \sin(\pi/2) - 2u \right) + \frac{2\pi - 2u}{2} - \frac{1}{4} \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \cos 2u + \frac{1}{2} \sin 2u \right]$$

$$= \frac{E_m^2}{2\pi} \left( \frac{1}{4} - \frac{3u}{4} + \frac{1}{4} \sin 2u \right)$$

for $u = 24.7^\circ,$

$$E_i = E_m \sqrt{\frac{3}{2\pi}} = 0.691 E_m$$
This compares with the value \( E_L = 0.707 \, E_m \) for sinusoidal supply. 

The power factor is therefore

\[
P F = \frac{P}{3E_m I_a} = \frac{7290}{3 \times 0.691 \times 400/\sqrt{3} \times \sqrt{2} \times 1.492} = 0.722
\]

This value is about 7% higher than the value 0.676 obtained with ideal supply.

**PROBLEMS**

**Three-Phase, Half-Wave Bridge Circuit with Resistive Load and Ideal Supply**

4.1 A set of balanced, three-phase sinusoidal voltages from an ideal supply is applied to a three-phase, half-wave bridge of ideal diodes with a resistive load (Fig. 4.2a). Sketch waveforms of the load current and supply current and show that the average load current is given by

\[
I_a = \frac{3\sqrt{3} \, E_m}{2\pi \, R}
\]

where \( E_m \) is the peak value of the supply phase voltage.

4.2 For the bridge of Problem 4.1, calculate the rms values \( I_L \) and \( I_a \) of the load and supply currents, respectively. Hence show that

\[
I_a = \frac{I_L}{\sqrt{3}}
\]

4.3 Calculate the operating power factor of the resistively loaded bridge of Problem 4.1 and show that it is independent of load resistance and supply voltage level.

4.4 Calculate values for the Fourier coefficients \( a_1 \) and \( b_1 \) of the fundamental (supply frequency) component of the line current in the bridge of Fig. 4.2a. Hence calculate the displacement factor.

4.5 For the bridge circuit of Fig. 4.2a calculate the rms value of the input current and also the rms value of its fundamental component. Hence calculate the supply current distortion factor. Use the value of the distortion together with the displacement factor (Problem 4.4) to calculate the power factor.

4.6 Derive expressions for the \( n \)th harmonic components \( a_n \) and \( b_n \) of the Fourier series representing the line current waveform \( i_a (\omega t) \) of Fig. 4.2. Show that \( a_n = 0 \) for \( n \) odd and \( b_n = 0 \) for \( n \) even.
4.7 A set of three-phase voltages of rms value 400 V at 50 Hz is applied to a half-wave diode bridge with a resistive load \( R = 40 \, \Omega \). Calculate the power transferred to the load.

4.8 In the three-phase, half-wave bridge of Fig. 4.2a deduce and sketch waveforms of the three supply currents and the load current if diode \( D_a \) fails to an open circuit.

4.9 Calculate the ripple factor for the supply current waveform \( i_a (\omega t) \), (Fig. 4.2d), obtained by resistive loading of a three-phase, half-wave diode bridge.

**Three-Phase Half-Wave Bridge Circuit with Highly Inductive Load and Ideal Supply**

4.10 A set of balanced, three-phase, sinusoidal voltages from an ideal supply is applied to a three-phase, half-wave diode bridge with a highly inductive load (Fig. 4.5a). Sketch waveforms of the supply voltages and currents and the load voltage and current. Show that the average load current retains the same value as with resistive load.

4.11 For the ideal, three-phase bridge of Problem 4.10, show that the rms value of the supply current \( I_s \) is related to the rms value of the load current \( I_L \) by

\[
I_s = \frac{I_L}{\sqrt{3}}
\]

4.12 Calculate the operating power factor of the inductively loaded bridge of Fig. 4.5a. Show that with a highly inductive load, the power factor is constant and independent of the values of \( L \) and \( R \). Compare the value of the power factor with that obtained when the load is purely resistive.

4.13 For a three-phase, half-wave bridge with highly inductive load calculate the average and rms values of the supply current. Hence show that the ripple factor of the supply current has a value \( RF = 1.41 \). How does this compare with the corresponding value for resistive load?

4.14 A set of three-phase voltages of rms line value 240 V at 50 Hz is applied to a three-phase, half-wave bridge of ideal diodes. The load consists of a resistor \( R = 10 \, \Omega \) in series with a large inductor. Calculate the power dissipation and compare this with the corresponding value in the absence of the (inductor) choke.

4.15 Calculate values for the Fourier coefficients \( a_1 \) and \( b_1 \) of the fundamental (supply frequency) component of the line current in the inductively loaded bridge of Fig. 4.5a. Hence calculate the displacement factor.
4.16 Calculate the magnitude and phase angle of the fundamental component of the line current for the inductively loaded bridge of Fig. 4.5a. Sketch this component for phase a together with the corresponding phase voltage and current.

4.17 For the three-phase bridge circuit of Fig. 4.5a calculate the rms value of the input current and also the rms value of its fundamental component. Hence calculate the supply current distortion factor. Use this value of the distortion factor together with the value of the displacement factor obtained from Problem 4.15 to determine the input power factor.

4.18 Derive expressions for the nth harmonic components of the Fourier series representing the line current \( i_a(t) \) of Fig. 4.5e. Compare the component terms, for particular values of \( n \), with corresponding values obtained for resistive load in Problem 4.6.

4.19 For an inductively loaded, three-phase, half-wave bridge the load voltage waveform is given in Fig. 4.5c. Calculate the Fourier series for this periodic waveform and show that its lowest ripple frequency is three times the supply frequency.

4.20 In the three-phase, half-wave bridge circuit of Fig. 4.5a, deduce and sketch waveforms of the three supply currents and the load current if diode \( D_a \) fails to an open circuit.

4.21 For the three-phase bridge in Problem 4.14 calculate the rms current and peak reverse voltage ratings required of the diodes.

**Three-Phase, Half-Wave Bridge Circuit with Highly Inductive Load in the Presence of Supply Inductance**

4.22 A set of balanced three-phase voltages is applied to an uncontrolled, three-phase, half-wave bridge with a highly inductive load. Each supply line has a series inductance \( L_s \) such that \( \omega L_s \) is about 10% of load resistor \( R \). Sketch the waveform of the load voltage and derive an expression for the average value in terms of the peak supply voltage per phase \( E_m \) and the overlap angle \( u \).

4.23 For the three-phase, half-wave diode bridge circuit with supply reactance, state an expression for the waveform of the supply voltage per phase. Show that the rms value of this is given by Eq. (4.50) and sketch its variation with \( u \) for \( 0 \leq u \leq 90^\circ \).

4.24 The overlap function \( \psi(u) \) for a three-phase, half-wave, diode bridge circuit is defined by Eq. (4.42). Sketch the variation of \( \psi(u) \) versus \( u \) in the range \( 0 \leq u \leq 90^\circ \).
4.25 The rms value $I_a$ of the supply current to a three-phase, half-wave, uncontrolled bridge rectifier with highly inductive load is related to the average load current $I_{av}$ by Eq. (4.41). Calculate the variation of $I_a$, assuming fixed $I_{av}$, for a range of values of overlap function $\psi(u)$. Sketch the per unit variation of $I_a$ with overlap angle $u$ for the range $0 \leq u \leq 90^\circ$.

4.26 A set of three-phase voltages of rms line value 240 V at 50 Hz is applied to a three-phase, uncontrolled, half-wave bridge. The load consists of a resistor $R = 10 \, \Omega$ in series with a large choke. Each supply line contains a series inductor $L_s$ of such value that $\omega L_s = 0.2R$. Calculate values of the rms voltage and current per phase at the bridge terminals and compare these with the values obtained with ideal supply.

4.27 For the bridge circuit of Problem 4.26 calculate the power dissipation and power factor. Compare the values with the respective values obtained with ideal supply.

4.28 The average load voltage $E_{av}$ for a three-phase, half-wave uncontrolled bridge is defined by Eq. (4.33). Calculate and sketch the variation of $E_{av}$ versus $u$ for the range $0 \leq u \leq 90^\circ$. Extend the sketch of $E_{av}$ for $u > 90^\circ$, using an appropriate relationship. Does the variation of $E_{av}$ versus $u$ indicate the change of mode of operation?

4.29 A three-phase, half-wave diode bridge circuit supplies power to a load resistor $R$ in series with a large inductor. The open-circuit supply voltages are a balanced set of sinusoidal voltages. Each supply line contains a series reactance $\omega L_s$, where $L_s << L$. Sketch waveforms of the supply phase, assuming mode I operation with an overlap angle $u \approx 20^\circ$. Show that during overlap the supply current varies sinusoidally.